91. Proposed by **F. P. MATZ, Sc. D., Ph. D.**, Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

There are two unequal square numbers the sum of whose sum, difference, product, and quotient, is a square. Find the two numbers.

Solution by **J. H. DRUMMOND, LL. D.**, Portland, Me.

Any two unequal squares answer the conditions of the question; for, let a^2 and b^2 be the numbers, then $a^2+b^2+a^2-b^2+a^2b^2+(a^2/b^2)$ must be a square. Reducing, and dropping the factor a^2, we have $2+b^2+(1/b^2)$, a square. This is readily put under the form $\dfrac{b^4+2b^2+1}{b^2}$, which is the square of $\dfrac{b^2+1}{b^2}$.

Q. E. D.

Also solved in a similar manner by *G. B. M. ZERR*, *J. SCHEFFER*, and *H. S. VANDIVER*.

92. Proposed by **L. C. WALKER. A.M.**, Professor of Mathematics, Petaluma High School, Petaluma, Cal.

(a) Find the least three integral numbers such that the difference of every two of them shall be a square number; (b) Find the least three square numbers such that the difference of every two of them shall be a square number.

Solution by **J. H. DRUMMOND, LL. D.**, Portland, Me.

Part 2. Let x^2, m^2x^2, and n^2x^2 be the three numbers; then m^2-1, n^2-1, and m^2-n^2 must all be squares.

Assume $m=\dfrac{p^2+1}{p^2-1}$ and $n=\dfrac{q^2+1}{q^2-1}$, and m^2-1 and n^2-1 will be squares and it remains to make $\left[\dfrac{p^2+1}{p^2-1}\right]^2-\left[\dfrac{q^2+1}{q^2-1}\right]=\square$, or reducing $(p^2q^2-1)(\dfrac{q^2}{p^2}-1)=\square$. This is done by making $pq=\dfrac{r^2+s^2}{2rs}$ and $\dfrac{q}{p}=\dfrac{t^2+u^2}{2tu}$.

But $pq\times q/p=q^2=\left[\dfrac{r^2+s^2}{2rs}\right]\left[\dfrac{t^2+u^2}{2tu}\right]$. Hence $rs(r^2+s^2)tu(t^2+u^2)$ must be a square. Take $r=f+g$, $s=f-g$, $t=h+k$, and $u=h-k$, and substituting and reducing, we have $(f^4-g^4)(h^4-k^4)=\square$. Assume $f^2=h^4-k^4+v^2$ and $g^2=h^4-k^4-v^2$ and $(f^4-g^4)=4v^2(h^4-k^4)$, and $(f^4-g^4)(h^4-k^4)$ becomes $4v^2(h^4-k^4)^2$, a square. Assume $h^2=a^2v^2$ and $k^4=b^2v^2$; then $h^2=av$ and $k^2=bv$. But $f^2-g^2=2v^2$. Assume $f+g=4v$ and $f-g=\frac{1}{2}v$, then $f=\frac{9}{4}v$ and $g=\frac{7}{4}v$. Then $r=4v$ and $s=\frac{1}{2}v$. Then $pq\left[=\dfrac{r^2+s^2}{2rs}\right]=\frac{65}{16}$, and $q/p=\dfrac{t^2+u^2}{2tu}$ $=\dfrac{2(h^2+k^2)}{2(h^2-k^2)}=\dfrac{a+b}{a-b}$; and $h^4-k^4+v^2=f^2=\dfrac{81v^2}{16}$, and $h^4-k^4=\dfrac{65v^2}{16}$. Therefore $a^2-b^2=\dfrac{h^4-k^4}{v^2}=\frac{65}{16}$. Now $pq\times q/p=q^2=\dfrac{65(a+b)}{16(a-b)}=\dfrac{65(a+b)^2}{16(a-b)^2}=(a+b)^2$ and $q=(a+b)$ and $p=(a-b)$. Hence $m=\dfrac{(a-b)^2+1}{(a-b)^2-1}$ and $n=\dfrac{(a+b)^2+1}{(a+b)^2-1}$, in which a and b may be any square numbers which make $a^2-b^2=\frac{65}{16}$.

It would seem that $a^2-b^2=\frac{65}{16}$ ought to lead readily to a general solution, but a and b were both so taken that they must be squares; b is readily found to